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DOUBLE EDGE-VERTEX DOMINATION NUMBER OF GRAPHS

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Abstract. An edge $e = uv$ of graph $G = (V, E)$ is said to edge-vertex dominate vertices u and v as well as all vertices adjacent to u and v. A set $S \subseteq E$ is called a double edge-vertex dominating set, if every vertex of V is edge-vertex dominated by at least two edges of S. The minimum cardinality of a double edge-vertex dominating set of G is the double edge-vertex domination number and is denoted $\gamma_{dev}(G)$. In this paper, we define double edge-vertex domination. Then we derive formulas for some special classes of graphs on double edge-vertex domination number.

Keywords: edge-vertex dominating set, vertex-edge dominating set, double edge-vertex dominating set, double vertex-edge dominating set.

AMS Subject Classification: 68R10, 68M10, 05C69.

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1 Introduction

In graph theory, various measurements are used to strengthen the stability of graphically modelable communication networks. These measures are called vulnerability measures. Domination which is one of these measures, has a wide range of applications.

Let $G = (V, E)$ be a simple graph. The set $N(v) = \{v \in V | uv \in E\}$ is open neighborhood and $N[v] = N(v) \cup N(u)$ is closed neigboorhood of $v \in V$ (Kulli, 2015). For any edge $e \in E$, the open edge neighborhood $N(e)$ of e is the set of edges adjacent to e (Kulli, 2016). For $S \subseteq V$ of vertices in a graph $G = (V, E)$ is called a dominating set if every vertex $v \in V$ is either an element of S or adjacent to an element of S. The minimum cardinality of a dominating set of G is called the domination number and is denoted $\gamma(G)$ (Haynes et al., 1998). A subset X of E is called an edge dominating set of G if every edge not in X is adjacent to some edge in X. The edge domination number $\gamma^{'}(G)$ of G is the minimum cardinality taken over all edge dominating sets of G. The concept of edge domination established by Mitchell & Hedetniemi (1977). A vertex v of $G = (V, E)$ is said to ve-dominate every edge incident to v, as well as every edge adjacent to these incident edges. A set $S \subseteq V$is a vertex-edge dominating set, if every edge of E is ve-dominated by at least one vertex of S. The minimum cardinality of a vertex-edge dominating set of G is the vertex-edge domination number and is denoted $\gamma_{ve}(G)$. Peters defined concept of vertex-edge domination inPeters (1986), then these concepts continued to be studied by Krishnakumari et al. (2014); Lewis et al. (2010); Lewis (2007); Boutrig et al. (2016). An edge $e = uv$ of graph $G = (V, E)$ is said to edge-vertex dominate vertices u and v as well as all vertices adjacent to u and v. A set $S \subseteq E$ is an edge-vertex dominating set, if every vertex of V is ev-dominated by at least an edge of S. The minimum cardinality of an edge-vertex dominating set of G is the edge-vertex domination number and is denoted by $\gamma_{ev}(G)$. Edge-vertex domination set and edge-vertex domination number were defined by Lewis

(2007).

In this paper, we define double edge-vertex domination of a graph which is denoted by $\gamma_{dev}(G)$.

2 Results on $\gamma_{dve}(G)$ of Binomial Trees and 2-ary Trees

Double vertex edge-domination concept was introduced by Krishnakumari et al. (2017). They established upper and lower bounds for some classes of graphs. They also determined exact values for paths and cycles. Then they characterized the trees depending on other types of domination. In this section we found double vertex edge domination number of binomial trees (Manuel et al., 2012) and 2-ary trees (West, 2001) and share their proofs.

Definition 1. *A subset $D \subseteq V$ is a double vertex-edge dominating set of G if every edge of E is vertex-edge dominated by at least two vertices of D. The double vertex-edge domination number of G, is the minimum cardinality of a double vertex-edge dominating set of G and denoted by denoted by $\gamma_{dve}(G)$. A double vertex-edge dominating set of G of minimum cardinality is called a $\gamma_{dve}(G)$-set by Krishnakumari et al. (2017).*

Theorem 1. *Let B_n be a binomial tree of order $n \geq 2$,*

$$\gamma_{dve}(B_n) = 2^{n-1}$$

Proof. Let's prove this theorem by mathematical induction.
For $n = 2$,

$$\gamma_{dve}(B_2) = 2^{2-1} = 2^1 = 2.$$

For $n = 3$,

$$\gamma_{dve}(B_3) = 2^{3-1} = 2^2 = 4.$$

Result is true for B_2 and B_3.

Our assumption asserts that

$$\gamma_{dve}(B_k) = 2^{k-1}$$

for $n = k$.
We want to show that for $n = k + 1$, double vertex-edge domination number of binomial tree is

$$\gamma_{dve}(B_{k+1}) = 2^{(k+1)-1}.$$

Number of vertices of B_{k+1} is twice as the vertices number of B_k. Since the number of vertices increases to exactly 2 times of vertices the B_k, we must select at least 2 times the number of vertices which were selected in B_k to dominate edges twice. Hence,

$$\gamma_{dve}(B_{k+1}) \geq \gamma_{dve}(B_k).2 = 2^{k-1}.2 = 2^{k-1}.2^1 = 2^{k-1+1} = 2^{(k+1)-1}$$

If we replace $k + 1$ by n we have,

$$= 2^{n-1}$$

□

Theorem 2. *Let H_n^2 be a 2-ary tree of order $n \geq 2$,*

$$\gamma_{dve}\left(H_n^2\right) = \begin{cases} 2 + 3.\sum_{i=0}^{\left\lfloor \frac{n}{4} \right\rfloor - 1} 2^{(n-2-4i)}, & if\ n\ mod\ 4 \equiv 1 \\ 3.\sum_{i=0}^{\left\lfloor \frac{n-1}{4} \right\rfloor} 2^{(n-2-4i)}, & otherwise \end{cases}$$

Proof. This proof is handled separately for 4 cases.

In all cases we need to start with the selection of 3 vertices to double ve-dominate all edges on the first and second levels. So, we select $2^{n-2}.3$ vertices at the beginning. The vertices which are selected at the beginning also double ve-dominate all the edges on the two levels above them. Therefore, we repeat these 3 vertices selection style for each H_2^2 at 2 levels above level 2^{n-4}. In this way, we start from 2^{n-2} and continue until $2^{(n-2)mod\ 4}$ by skipping 4 levels. Thus, i starts from 0 and continue until $2^{n-2-4i} = 2^{(n-2)mod\ 4}$. The number of vertices at each level is twice as the number of vertices at the level under itself.

Since we do operations in $mod\ 4$, we should examine n separately for $n\ mod\ 4 \equiv 2$, $n\ mod\ 4 \equiv 3$, $n\ mod\ 4 \equiv 0$ and $n\ mod\ 4 \equiv 1$. So we take n as $4k+2$ for $n\ mod\ 4 \equiv 2$, $4k+3$ for $n\ mod\ 4 \equiv 3$, $4k$ for $n\ mod\ 4 \equiv 0$, $4k+1$ for $n\ mod\ 4 \equiv 1$ respectively.

- **Case 1:** For $n = 4k+2$, $n\ mod\ 4 \equiv 2$, result is true for $n = 2$.

$$\gamma_{dve}(H_2^2) = 3.\sum_{i=0}^{\lfloor \frac{n-1}{4} \rfloor} 2^{(n-2-4i)}$$

$$= 3.\sum_{i=0}^{\lfloor \frac{2-1}{4} \rfloor} 2^{(2-2-4i)}$$

$$= 3.\sum_{i=0}^{\lfloor \frac{1}{4} \rfloor} 2^{(0-4i)}$$

$$= 3.\sum_{i=0}^{0} 2^{(0-4i)}$$

$$= 3.2^{0-4.0} = 3.2^0 = 3.1 = 3.$$

Our assumptation asserts that,

$$\gamma_{dve}(H_{4k+2}^2) = 3.\sum_{i=0}^{\lfloor \frac{4k+2-1}{4} \rfloor} 2^{4k+2-2-4i}$$

$$= 3.\sum_{i=0}^{\lfloor \frac{4k+1}{4} \rfloor} 2^{4k-4i}$$

$$= 3.2^{4k-4.0} + 3.2^{4k-4.1} + ... + 3.2^{4k-4.\lfloor \frac{4k+2-1}{4} \rfloor}$$
$$= 3.2^{4k} + 3.2^{4k-4} + ... + 3.2^{4k-4.\lfloor \frac{4k+1}{4} \rfloor}.$$

We want to prove that for $n = 4k+6$, double vertex-edge domination number of 2-ary tree is

$$\gamma_{dve}(H_{4k+6}^2) = 3.\sum_{i=0}^{\lfloor \frac{4k+6-1}{4} \rfloor} 2^{4k+6-2-4i}$$

$$= 3.\sum_{i=0}^{\lfloor \frac{4k+5}{4} \rfloor} 2^{4k+4-4i}.$$

Since we add 4 levels to H^2_{4k+2}, the number of vertices of H^2_{4k+6} increases to 2^4 times of the vertices of the H^2_{4k+2} so we do operations in mod 4,

$$\gamma_{dve}(H^2_{4k+6}) \geq \gamma_{dve}(H^2_{4k+2}) + 3.2^{4k-4\lfloor\frac{4k+1}{4}\rfloor}.2^1$$

$$= 3.\sum_{i=0}^{\lfloor\frac{4k+1}{4}\rfloor} 2^{4k-4i} + 3.2^{4k-4\lfloor\frac{4k+1}{4}\rfloor}.2^1$$

$$= 3.2^{4k-4.0} + 3.2^{4k-4.1} + ... + 3.2^{4k-4\lfloor\frac{4k+1}{4}\rfloor} + 3.2^{4k-4\lfloor\frac{4k+1}{4}\rfloor}.2^1$$

$$= 3.2^{4k} + 3.2^{4k-4} + ... + 3.2^{4k-4\lfloor\frac{4k+1}{4}\rfloor} + 3.2^{4k-4\lfloor\frac{4k+2}{4}\rfloor+1}$$

$$\geq 3.2^{4k} + 3.2^{4k-4} + ... + 3.2^{4k-4\lfloor\frac{4k+1}{4}\rfloor} + 3.2^{4k-4\lfloor\frac{4k+1}{4}+1\rfloor}$$

$$= 3.2^{4k} + 3.2^{4k-4} + ... + 3.2^{4k-4\lfloor\frac{4k+1}{4}\rfloor} + 3.2^{4k-4\lfloor\frac{4k+1+4}{4}\rfloor}$$

$$= 3.2^{4k} + 3.2^{4k-4} + ... + 3.2^{4k-4.\lfloor\frac{4k+1}{4}\rfloor} + 3.2^{4k-4\lfloor\frac{4k+5}{4}\rfloor}$$

$$= \sum_{i=0}^{\lfloor\frac{4k+5}{4}\rfloor} 2^{4k+4-4i}$$

$$= \sum_{i=0}^{\lfloor\frac{4k+6-1}{4}\rfloor} 2^{4k+6-2-4i}.$$

- **Case 2:** For $n = 4k+3$, $n\ mod\ 4 \equiv 3$, result is true for $n = 3$.

$$\gamma_{dve}(H^2_3) = 3.\sum_{i=0}^{\lfloor\frac{n-1}{4}\rfloor} 2^{(n-2-4i)}$$

$$= 3.\sum_{i=0}^{\lfloor\frac{3-1}{4}\rfloor} 2^{(3-2-4i)}$$

$$= 3.\sum_{i=0}^{\lfloor\frac{2}{4}\rfloor} 2^{(1-4i)}$$

$$= 3.\sum_{i=0}^{0} 2^{(1-4i)} = 3.2^{1-4.0} = 3.2^1 = 3.2 = 6.$$

Our assumptation asserts that,

$$\gamma_{dve}(H^2_{4k+3}) = 3.\sum_{i=0}^{\lfloor\frac{4k+3-1}{4}\rfloor} 2^{4k+3-2-4i}$$

$$= 3.\sum_{i=0}^{\lfloor\frac{4k+2}{4}\rfloor} 2^{4k+1-4i}$$

$$= 3.2^{4k+1-4.0} + 3.2^{4k+1-4.1} + ... + 3.2^{4k+1-4.\lfloor\frac{4k+3-1}{4}\rfloor}$$

$$= 3.2^{4k+1} + 3.2^{4k-3} + ... + 3.2^{4k+1-4.(\frac{4k+2}{4})}.$$

We want to prove that for $n = 4k+7$, double vertex-edge domination number of 2-ary tree is

$$\gamma_{dve}(H^2_{4k+7}) = 3. \sum_{i=0}^{\lfloor \frac{4k+7-1}{4} \rfloor} 2^{4k+7-2-4i}$$

$$= 3. \sum_{i=0}^{\lfloor \frac{4k+6}{4} \rfloor} 2^{4k+5-4i}.$$

Since we add 4 levels to H^2_{4k+3}, the number of vertices of H^2_{4k+7} increases to 2^4 times of the vertices of the H^2_{4k+3} so we do operations in mod 4,

$$\gamma_{dve}(H^2_{4k+7}) \geq \gamma_{dve}(H^2_{4k+3}) + 3.2^{4k+1-4\lfloor \frac{4k+2}{4} \rfloor}.2^1$$

$$= 3. \sum_{i=0}^{\lfloor \frac{4k+2}{4} \rfloor} 2^{4k+1-4i} + 3.2^{4k+1-4\lfloor \frac{4k+2}{4} \rfloor}.2^1$$

$$= 3.2^{4k+1-4.0} + 3.2^{4k+1-4.1} + ... + 3.2^{4k+1-4\lfloor \frac{4k+2}{4} \rfloor} + 3.2^{4k+1-4\lfloor \frac{4k+2}{4} \rfloor}.2^1$$

$$= 3.2^{4k+1} + 3.2^{4k-3} + ... + 3.2^{4k+1-4\lfloor \frac{4k+2}{4} \rfloor} + 3.2^{4k+1-4(\lfloor \frac{4k+2}{4} \rfloor+1)}$$

$$\geq 3.2^{4k+1} + 3.2^{4k-3.1} + ... + 3.2^{4k+1-4\lfloor \frac{4k+2}{4} \rfloor} + 3.2^{4k+1-4(\lfloor \frac{4k+2}{4}+1 \rfloor}$$

$$= 3.2^{4k+1-4.0} + 3.2^{4k+1-4.1} + ... + 3.2^{4k+1-4.\lfloor \frac{4k+2}{4} \rfloor} + 3.2^{4k+1-4\lfloor \frac{4k+2+4}{4} \rfloor}$$

$$= 3.2^{4k+1} + 3.2^{4k-3} + ... + 3.2^{4k+1-4\lfloor \frac{4k+2}{4} \rfloor} + 3.2^{4k+1-4\lfloor \frac{4k+6}{4} \rfloor}$$

$$= \sum_{i=0}^{\lfloor \frac{4k+6}{4} \rfloor} 2^{4k+5-4i}$$

$$= \sum_{i=0}^{\lfloor \frac{4k+7-1}{4} \rfloor} 2^{4k+7-2-4i}.$$

- **Case 3:** For $n = 4k$, n mod $4 \equiv 0$, result is true for $n = 4$.

$$\gamma_{dve}(H^2_4) = 3. \sum_{i=0}^{\lfloor \frac{n-1}{4} \rfloor} 2^{(n-2-4i)}$$

$$= 3. \sum_{i=0}^{\lfloor \frac{4-1}{4} \rfloor} 2^{(4-2-4i)}$$

$$= 3. \sum_{i=0}^{\lfloor \frac{3}{4} \rfloor} 2^{(2-4i)}$$

$$= 3. \sum_{i=0}^{0} 2^{(2-4i)}$$

$$= 3.2^{2-4.0} = 3.2^2 = 3.4 = 12.$$

Our assumption asserts that,

$$\gamma_{dve}(H^2_{4k}) = 3.\sum_{i=0}^{\lfloor\frac{4k-1}{4}\rfloor} 2^{4k-2-4i}$$

$$= 3.2^{4k-2-4.0} + 3.2^{4k-2-4.1} + ... + 3.2^{4k-2-4.\lfloor\frac{4k-1}{4}\rfloor}$$

$$= 3.2^{4k-2} + 3.2^{4k-6} + ... + 3.2^{4k-2-4.\lfloor\frac{4k-1}{4}\rfloor}$$

We want to prove that for $n = 4k+4$, double vertex-edge domination number of 2-ary tree is

$$\gamma_{dev}(H^2_{4k+4}) = 3.\sum_{i=0}^{\lfloor\frac{4k+4-1}{4}\rfloor} 2^{4k+4-2-4i}$$

$$= 3.\sum_{i=0}^{\lfloor\frac{4k+3}{4}\rfloor} 2^{4k+2-4i}.$$

Since we add 4 levels to H^2_{4k}, the number of vertices of H^2_{4k+4} increases to 2^4 times of the vertices of the H_{4k} but we do operations in mod 4,

$$\gamma_{dev}(H^2_{4k+4}) \geq \gamma_{dev}(H^2_{4k}) + 3.2^{4k-2-4\lfloor\frac{4k-1}{4}\rfloor}.2^1$$

$$= 3.\sum_{i=0}^{\lfloor\frac{4k-1}{4}\rfloor} 2^{4k-2-4i} + 3.2^{4k-2-4\lfloor\frac{4k-1}{4}\rfloor}.2^1$$

$$= 3.2^{4k-2-4.0} + 3.2^{4k-2-4.1} + ... + 3.2^{4k-2-4.\lfloor\frac{4k-1}{4}\rfloor} + 3.2^{4k-2-4\lfloor\frac{4k-1}{4}\rfloor}.2^1$$

$$= 3.2^{4k-2} + 3.2^{4k-6} + ... + 3.2^{4k-2-4\lfloor\frac{4k-1}{4}\rfloor} + 3.2^{4k-2-4\lfloor\frac{4k-1}{4}\rfloor+1}$$

$$\geq 3.2^{4k-2} + 3.2^{4k-6} + ... + 3.2^{4k-2-4.\lfloor\frac{4k-1}{4}\rfloor} + 3.2^{4k-2-4\lfloor\frac{4k-1}{4}+1\rfloor}$$

$$= 3.2^{4k-2-4.0} + 3.2^{4k-2-4.1} + ... + 3.2^{4k-2-4.\lfloor\frac{4k-1}{4}\rfloor} + 3.2^{4k-2-4\lfloor\frac{4k-1+4}{4}\rfloor}$$

$$= 3.2^{4k-2} + 3.2^{4k-6} + ... + 3.2^{4k-2-4.\lfloor\frac{4k-1}{4}\rfloor} + 3.2^{4k-2-4\lfloor\frac{4k+3}{4}\rfloor}$$

$$= \sum_{i=0}^{\lfloor\frac{4k+3}{4}\rfloor} 2^{4k+2-4i}$$

$$= \sum_{i=0}^{\lfloor\frac{4k+4-1}{4}\rfloor} 2^{4k+4-2-4i}.$$

- **Case 4:** For $n = 4k + 1$, $n \; mod \; 4 \equiv 1$, result is true for $n = 5$.

$$\gamma_{dve}(H^2_5) = 2 + 3.\sum_{i=0}^{\lfloor\frac{n}{4}\rfloor-1} 2^{(n-2-4i)}$$

$$= 2 + 3.\sum_{i=0}^{\lfloor\frac{5}{4}\rfloor-1} 2^{(5-2-4i)}$$

$$= 2 + 3. \sum_{i=0}^{\lfloor \frac{5}{4} \rfloor - 1} 2^{(3-4i)}$$

$$= 2 + 3. \sum_{i=0}^{0} 2^{(3-4i)}$$

$$= 2 + 3.2^{3-4.0} = 2 + 3.2^3 = 2 + 3.8 = 2 + 24 = 26.$$

Our assumptation asserts that,

$$\gamma_{dve}(H^2_{4k+1}) = 2 + 3. \sum_{i=0}^{\lfloor \frac{4k+1}{4} \rfloor - 1} 2^{4k+1-2-4i}$$

$$= 2 + 3. \sum_{i=0}^{\lfloor \frac{4k+1}{4} \rfloor - 1} 2^{4k-1-4i}$$

$$= 2 + 3.2^{4k-1-4.0} + 3.2^{4k-1-4.1} + ... + 3.2^{4k-1-4.(\lfloor \frac{4k+1}{4} \rfloor - 1)}$$
$$= 2 + 3.2^{4k-1} + 3.2^{4k-5} + ... + 3.2^{4k-1-4(\lfloor \frac{4k+1}{4} \rfloor - 1)}.$$

We want to prove that for $n = 4k+5$, double vertex-edge domination number of 2-ary tree is

$$\gamma_{dve}(H^2_{4k+5}) = 2 + 3. \sum_{i=0}^{\lfloor \frac{4k+5}{4} \rfloor - 1} 2^{4k+5-2-4i}$$

$$= 2 + 3. \sum_{i=0}^{\lfloor \frac{4k+5}{4} \rfloor - 1} 2^{4k+3-4i}.$$

Since we add 4 levels to H^2_{4k+1}, the number of vertices of H^2_{4k+5} increases to 2^4 times of the vertices of the H^2_{4k+1} so we do operations in mod 4,

$$\gamma_{dve}(H^2_{4k+5}) \geq \gamma_{dve}(H^2_{4k+1}) + 3.2^{4k-1-4.(\lfloor \frac{4k+1}{4} \rfloor - 1)}.2^1$$

$$= 2 + 3. \sum_{i=0}^{\lfloor \frac{4k+1}{4} \rfloor - 1} 2^{4k-1-4i} + 3.2^{4k-1-4(\lfloor \frac{4k+1}{4} \rfloor - 1)}.2^1$$

$$= 2 + 3.2^{4k-1-4.0} + 3.2^{4k-1-4.1} + ... + 3.2^{4k-1-4.(\lfloor \frac{4k+1}{4} \rfloor - 1)} + 3.2^{4k-1-4(\lfloor \frac{4k+1}{4} \rfloor - 1)}.2^1$$
$$= 2 + 3.2^{4k-1} + 3.2^{4k-5} + ... + 3.2^{4k-1-4.(\lfloor \frac{4k+1}{4} \rfloor - 1)} + 3.2^{4k-1-4(\lfloor \frac{4k+1}{4} \rfloor - 1)+1}$$
$$\geq 2 + 3.2^{4k-1} + 3.2^{4k-5} + ... + 3.2^{4k-1-4.(\lfloor \frac{4k+1}{4} \rfloor - 1)} + 3.2^{4k-1-4(\lfloor \frac{4k+1}{4} + 1 \rfloor)-1}$$
$$= 2 + 3.2^{4k-1} + 3.2^{4k-5} + ... + 3.2^{4k-1-4(\lfloor \frac{4k+1}{4} \rfloor - 1)} + 3.2^{4k-1-4(\lfloor \frac{4k+1+4}{4} \rfloor - 1)}$$
$$= 2 + 3.2^{4k-1} + 3.2^{4k-5} + ... + 3.2^{4k-1-4(\lfloor \frac{4k+1}{4} \rfloor - 1)} + 3.2^{4k-1-4(\lfloor \frac{4k+5}{4} \rfloor - 1)}$$

$$= 2 + 3. \sum_{i=0}^{(\lfloor \frac{4k+5}{4} \rfloor - 1)} 2^{4k-3-4i}$$

$$= \sum_{i=0}^{\lfloor \frac{4k+5}{4} \rfloor - 1} 2^{4k+5-2-4i}.$$

$\square$

3 Double Edge-Vertex Domination and Results on P_n, C_n,K_n,B_n,$K_{m,n}$, H_n^2

In this section, we define double edge-vertex dominating set and double edge-vertex domination number, we give resultson double edge-vertex domination number for some classes of graphs and we prove these results .

Definition 2. *An edge $e = uv$ of graph $G = (V, E)$ is said to ev-dominate vertices u and v as well as all vertices adjacent to u and v. A set $S \subseteq E$ is a double edge-vertex dominating set, if every vertex of V is ev-dominated by at least two edges of S. The minimum cardinality of a double edge-vertex dominating set of G is the double edge-vertex domination number and is denoted $\gamma_{dev}(G)$.*

Theorem 3. *Let P_n be a path graph of order $n \geq 3$,*

$$\gamma_{dev}(P_n) = \begin{cases} \lceil \frac{n}{2} \rceil, & if\ (n-1) mod\ 4 \equiv 2 \\ \lceil \frac{n}{2} \rceil + 1, & otherwise \end{cases}$$

Proof. Let's prove this theorem by mathematical induction.

Since we do operations in mod 4, we should examine n separately for $(n-1)\ mod\ 4 \equiv 2$, $(n-1)\ mod\ 4 \equiv 3$, $(n-1)\ mod\ 4 \equiv 0$ and $(n-1)\ mod\ 4 \equiv 1$. So we take n as $4k+3$ for $(n-1)\ mod\ 4 \equiv 2$, $4k$ for $(n-1)\ mod\ 4 \equiv 3$, $4k+1$ for $(n-1)\ mod\ 4 \equiv 0$, $4k+2$ for $(n-1)\ mod\ 4 \equiv 1$ respectively.

- For $n = 4k+3$, $(n-1) mod\ 4 \equiv 2$, result is true for $n = 3$,

$$\gamma_{dev}(P_3) = \left\lceil \frac{3}{2} \right\rceil = 2.$$

Our assumption asserts that,

$$\gamma_{dev}(P_{4k+3}) = \left\lceil \frac{4k+3}{2} \right\rceil$$

for $(4k+2) mod\ 4 \equiv 2$.
We want to prove that for $n = 4k+3$, double edge-vertex domination number of path graph is

$$\gamma_{dev}(P_{4k+3}) = \left\lceil \frac{4k+3}{2} \right\rceil.$$

We added 4 vertices to P_{4k+3}. These 4 vertices ev-dominated by at least 2 edges. Hence,

$$\begin{aligned} \gamma_{dev}(P_{4k+7}) &\geq \gamma_{dev}(P_{4k+3}) + 2 \\ &= \left\lceil \frac{4k+3}{2} \right\rceil + \frac{4}{2} \\ &\geq \left\lceil \frac{4k+3}{2} + \frac{4}{2} \right\rceil \\ &= \left\lceil \frac{4k+7}{2} \right\rceil. \end{aligned}$$

- For $n = 4k$, $(n-1) mod\ 4 \equiv 3$, result is true for $n = 4$,

$$\gamma_{dev}(P_4) = \left\lceil \frac{4}{2} \right\rceil + 1 = 2 + 1 = 3.$$

Our assumption asserts that,

$$\gamma_{dev}(P_{4k}) = \left\lceil \frac{4k}{2} \right\rceil + 1$$

for $(4k-1) mod\ 4 = 3$.
We want to prove that for $n = 4k + 4$, double edge-vertex domination number of path graph is

$$\gamma_{dev}(P_{4k+4}) = \left\lceil \frac{4k+4}{2} \right\rceil + 1.$$

We added 4 vertices to P_{4k}. These 4 vertices ev-dominated by at least 2 edges. Hence,

$$\begin{aligned} \gamma_{dev}(P_{4k+4}) &\geq \gamma_{dev}(P_{4k}) + 2 \\ &= (\left\lceil \frac{4k}{2} \right\rceil + 1) + 2 \\ &= \left\lceil \frac{4k}{2} \right\rceil + 1 + 2 = \left\lceil \frac{4k}{2} \right\rceil + 2 + 1 \\ &= \left\lceil \frac{4k}{2} \right\rceil + \frac{4}{2} + 1 \\ &\geq \left\lceil \frac{4k+4}{2} \right\rceil + 1 \\ &= \left\lceil \frac{4k+4}{2} \right\rceil + 1. \end{aligned}$$

- For $n = 4k + 1$, $(n-1) mod\ 4 \equiv 0$, result is true for $n = 5$,

$$\gamma_{dev}(P_5) = \left\lceil \frac{5}{2} \right\rceil + 1 = 3 + 1 = 4.$$

Our assumption asserts that,

$$\gamma_{dev}(P_{4k+1}) = \left\lceil \frac{4k+1}{2} \right\rceil + 1$$

for $(4k) mod\ 4 = 0$.
We want to prove that for $n = 4k + 5$, double edge-vertex domination number of path graph is

$$\gamma_{dev}(P_{4k+5}) = \left\lceil \frac{4k+5}{2} \right\rceil + 1.$$

We added 4 vertices to P_{4k+1}. These 4 vertices ev-dominated by at least 2 edges. Hence, *

$$\begin{aligned} \gamma_{dev}(P_{4k+5}) &\geq \gamma_{dev}(P_{4k+1}) + 2 \\ &= (\left\lceil \frac{4k+1}{2} \right\rceil + 1) + 2 \end{aligned}$$

$$= \left\lceil \frac{4k+1}{2} \right\rceil + 1 + 2$$

$$= \left\lceil \frac{4k+1}{2} \right\rceil + 2 + 1$$

$$= \left\lceil \frac{4k+1}{2} \right\rceil + \frac{4}{2} + 1$$

$$\geq \left\lceil \frac{4k+1+4}{2} \right\rceil + 1$$

$$= \left\lceil \frac{4k+5}{2} \right\rceil + 1.$$

- For $n = 4k+2$, $(n-1) mod\ 4 = 1$, result is true for $n = 6$,

$$\gamma_{dev}(P_6) = \left\lceil \frac{6}{2} \right\rceil + 1 = 3 + 1 = 4.$$

Our assumption asserts that,

$$\gamma_{dev}(P_{4k+2}) = \left\lceil \frac{4k+2}{2} \right\rceil + 1$$

for $(4k+1) mod\ 4 = 1$.
We want to prove that for $n = 4k+6$, double edge-vertex domination number of path graph is

$$\gamma_{dev}(P_{4k+6}) = \left\lceil \frac{4k+6}{2} \right\rceil + 1.$$

We added 4 vertices to P_{4k+2}. These 4 vertices ev-dominated by at least 2 edges. Hence,

$$\gamma_{dev}(P_{4k+6}) \geq \gamma_{dev}(P_{4k+2}) + 2$$

$$= (\left\lceil \frac{4k+2}{2} \right\rceil + 1) + 2$$

$$= \left\lceil \frac{4k+2}{2} \right\rceil + 1 + 2$$

$$= \left\lceil \frac{4k+2}{2} \right\rceil + 2 + 1$$

$$= \left\lceil \frac{4k+2}{2} \right\rceil + \frac{4}{2} + 1$$

$$\geq \left\lceil \frac{4k+2+4}{2} \right\rceil + 1$$

$$= \left\lceil \frac{4k+6}{2} \right\rceil + 1.$$

□

Theorem 4. *Let C_n be a cycle graph of order $n \geq 3$,*

$$\gamma_{dev}(C_n) = \begin{cases} \lceil \frac{n}{2} \rceil + 1, & if\ n\ mod\ 4 \equiv 2 \\ \lceil \frac{n}{2} \rceil, & otherwise \end{cases}$$

Proof. We use mathematical induction for proof of this theorem.
Since we do operations in mod 4, we should examine n separately for $n \ mod \ 4 \equiv 3$, $n \ mod \ 4 \equiv 0$, $n \ mod \ 4 \equiv 1$ and $n \ mod \ 4 \equiv 2$. So we take n as $4k+3$ for $n \ mod \ 4 \equiv 3$, $4k$ for $n \ mod \ 4 \equiv 0$, $4k+1$ for $n \ mod \ 4 \equiv 1$, $4k+2$ for $n \ mod \ 4 \equiv 2$ respectively.

- For $n = 4k+3$, $n \ mod \ 4 \equiv 3$, result is true for $n = 3$,

$$\gamma_{dev}(C_3) = \left\lceil \frac{3}{2} \right\rceil = 2.$$

Our assumption asserts that,

$$\gamma_{dev}(C_{4k+3}) = \left\lceil \frac{4k+3}{2} \right\rceil$$

for $n \ mod \ 4 = 3$.
We want to prove that for $n = 4k+7$, double edge-vertex domination number of cycle graph is

$$\gamma_{dev}(C_{4k+7}) = \left\lceil \frac{4k+7}{2} \right\rceil.$$

We added 4 vertices to C_{4k+3}. These 4 vertices ev-dominated by at least 2 edges. Hence,

$$\begin{aligned}
\gamma_{dev}(C_{4k+7}) &\geq \gamma_{dev}(C_{4k+3}) + 2 \\
&= (\left\lceil \frac{4k+3}{2} \right\rceil) + 2 \\
&= \left\lceil \frac{4k+3}{2} \right\rceil + \frac{4}{2} \\
&\geq \left\lceil \frac{4k+3}{2} + \frac{4}{2} \right\rceil \\
&= \left\lceil \frac{4k+3+4}{2} \right\rceil \\
&= \left\lceil \frac{4k+7}{2} \right\rceil.
\end{aligned}$$

- For $n = 4k$, $n \ mod \ 4 \equiv 0$, result is true for $n = 4$,

$$\gamma_{dev}(C_4) = \left\lceil \frac{4}{2} \right\rceil = 2.$$

Our assumption asserts that,

$$\gamma_{dev}(C_{4k}) = \left\lceil \frac{4k}{2} \right\rceil$$

for $n \ mod \ 4 = 0$.
We want to prove that for $n = 4k+4$, double edge-vertex domination number of cycle graphs is

$$\gamma_{dev}(C_{4k+4}) = \left\lceil \frac{4k+4}{2} \right\rceil.$$

We added 4 vertices to C_{4k}. These 4 vertices ev-dominated by at least 2 edges. Hence,

$$\gamma_{dev}(C_{4k+4}) \geq \gamma_{dev}(C_{4k}) + 2$$

$$= (\left\lceil \frac{4k}{2} \right\rceil) + 2$$

$$= \left\lceil \frac{4k}{2} \right\rceil + \frac{4}{2}$$

$$\geq \left\lceil \frac{4k}{2} + \frac{4}{2} \right\rceil$$

$$= \left\lceil \frac{4k+4}{2} \right\rceil.$$

- For $n = 4k+1$, $n\ mod\ 4 \equiv 1$, result is true for $n = 5$,

$$\gamma_{dev}(C_5) = \left\lceil \frac{5}{2} \right\rceil = 3.$$

Our assumptation asserts that,

$$\gamma_{dev}(C_{4k+1}) = \left\lceil \frac{4k+1}{2} \right\rceil$$

for $n\ mod\ 4 = 1$.
We want to prove that for $n = 4k+5$, double edge-vertex domination number of cycle graph is

$$\gamma_{dev}(C_{4k+5}) = \left\lceil \frac{4k+5}{2} \right\rceil.$$

We added 4 vertices to C_{4k+1}. These 4 vertices ev-dominated by at least 2 edges. Hence,

$$\gamma_{dev}(C_{4k+5}) \geq \gamma_{dev}(C_{4k+1}) + 2$$

$$= (\left\lceil \frac{4k+1}{2} \right\rceil) + 2$$

$$= \left\lceil \frac{4k+1}{2} \right\rceil + \frac{4}{2}$$

$$\geq \left\lceil \frac{4k+1}{2} + \frac{4}{2} \right\rceil$$

$$= \left\lceil \frac{4k+5}{2} \right\rceil.$$

- For $n = 4k+2$, $n\ mod\ 4 \equiv 2$, result is true for $n = 6$,

$$\gamma_{dev}(C_6) = \left\lceil \frac{6}{2} \right\rceil + 1 = 4.$$

Our assumptation asserts that,

$$\gamma_{dev}(C_{4k+2}) = \left\lceil \frac{4k+2}{2} \right\rceil + 1$$

for $n\ mod\ 4 = 2$. We want to prove that for $n = 4k+6$, double edge-vertex domination number of cycle graph is

$$\gamma_{dev}(C_{4k+6}) = \left\lceil \frac{4k+6}{2} \right\rceil + 1.$$

We added 4 vertices to C_{4k+2}. These 4 vertices ev-dominated by at least 2 edges. Hence,

$$\begin{aligned}\gamma_{dev}(C_{4k+6}) &\geq \gamma_{dev}(C_{4k+2}) + 2\\ &= (\left\lceil \frac{4k+2}{2} \right\rceil + 1) + 2\\ &= \left\lceil \frac{4k+2}{2} \right\rceil + 1 + 2\\ &= \left\lceil \frac{4k+2}{2} \right\rceil + 2 + 1\\ &= \left\lceil \frac{4k+2}{2} \right\rceil + \frac{4}{2} + 1\\ &\geq \left\lceil \frac{4k+2+4}{2} \right\rceil + 1\\ &= \left\lceil \frac{4k+6}{2} \right\rceil + 1.\end{aligned}$$

□

Theorem 5. *Let K_n be a complete graph of order $n \geq 3$,*

$$\gamma_{dev}(K_n) = 2$$

Proof. Since there are edges between all vertex pairs in complete graph, any edge which we choose dominates all vertices. So any two edges which we choose dominate all the vertices.Then

$$\gamma_{dev}(K_n) = 2.$$

□

Theorem 6. *Let $K_{m,n}$ be a complete bipartite graph of order $m \times n$ and $m, n \geq 2$,*

$$\gamma_{dev}\ (K_{m,n})\ {=}2$$

Proof. The distance among two vertices is at most 2 regardless of the number of vertices in complete bipartite graph. Any edge which we choose dominates all vertices in this graph. In this case, any selected two edges dominate all vertices twice. So,

$$\gamma_{dev}(K_{m,n}) = 2.$$

□

Theorem 7. *Let $K_{n_1,n_2,\ldots,n_r}$ be a complete r-partite graph of order $n_1 \times n_2 \times \ldots \times n_r$,*

$$\gamma_{dev}\ (K_{n_1,n_2,\ldots,n_r})\ {=}2$$

Proof. The distance among two vertices is at most 2 regardless of the number of vertices in complete r-partite graph. Any edge which we choose dominates all vertices in this graph.In this case, any selected two edges dominate all vertices twice.So,

$$\gamma_{dev}(K_{n_1,n_2,\ldots,n_r}) = 2.$$

□

Theorem 8. *Let B_n be a binomial tree of order $n \geq 3$,*

$$\gamma_{dev}(B_n) = 5.2^{n-3}$$

Proof. Let's prove this theorem by mathematical induction. For $n = 3$, $\gamma_{dev}(B_3) = 5$ and we must choose one of these 5 edges which connects two B_2.
Our assumption asserts that if $n = k$,

$$\gamma_{dev}(B_k) = 5.2^{k-3}.$$

We want to show that double edge-vertex domination number of binomial tree is

$$\gamma_{dev}(B_{k+1}) = 5.2^{(k+1)-3}$$

for $n = k + 1$.
Number of vertices of B_{k+1} is twice as the number of the vertices of B_k. Since the number of vertices increases exactly 2 times of the number of vertices of B_k, each time we must select 2 times of the number of edges which were selected in B_k to dominate vertices twice. Hence,

$$\begin{aligned}\gamma_{dev}(B_{k+1}) &\geq \gamma_{dev}(B_k).2\\ &= 5.2^{k-3}.2\\ &= 5.2^{k-3+1}\\ &= 5.2^{(k+1)-3}.\end{aligned}$$

If we replace $k + 1$ by n, we have

$$= 5.2^{n-3}.$$

□

Theorem 9. *Let H_n^2 be a 2-ary tree of order n,*

$$\gamma_{dev}\left(H_n^2\right) = \begin{cases} 2+\sum_{i=1}^{\frac{n}{4}} 2^{4i}, & if\ n\ mod\ 4 \equiv 0 \\ \sum_{i=0}^{\left\lfloor \frac{n}{4} \right\rfloor} 2^{n(mod\ 4)}.2^{4i}, & otherwise \end{cases}$$

Proof. This proof is done in 4 cases. In all cases we need to select 2^n edges to double ev-dominate all vertices on the first and second levels at the beginning. The edges which are selected at the beginning also double ev-dominate all the vertices on the first level above them. Also, the edges which will be newly selected double ev-dominate all the vertices on the first level under them. Therefore, we make this selection for every 4 levels. In this way, we begin with 2^n and continue until $2^{(nmod\ 4)}$. Thus, i starts from 0 and continue until $2^{(nmod\ 4)+4i} = 2^n$.
The number of vertices at each level is twice as the number of vertices at the level under itself. Since we do operations in mod 4, we should examine n separately for $n\ mod\ 4 \equiv 1$, $n\ mod\ 4 \equiv 2$, $n\ mod\ 4 \equiv 3$ and $n\ mod\ 4 \equiv 0$. So we take n as $4k+1$ for $n\ mod\ 4 \equiv 1$, $4k+2$ for $n\ mod\ 4 \equiv 2$, $4k + 3$ for $n\ mod\ 4 \equiv 3$, $4k$ for $n\ mod\ 4 \equiv 0$ respectively.

- **Case 1:** For $n = 4k + 1$, $n\ mod\ 4 \equiv 1$, result is true for $n = 1$.

$$\gamma_{dev}(H_1^2) = \sum_{i=0}^{\left\lfloor \frac{1}{4} \right\rfloor} 2^{1(mod\ 4)}.2^{4i} = 2^1.2^0 = 2^1 = 2.$$

Our assumptation asserts that,

$$\gamma_{dev}(H_{4k+1}^2) = \sum_{i=0}^{\left\lfloor \frac{4k+1}{4} \right\rfloor} 2^{4k+1(mod\ 4)}.2^{4i}$$

$$= \sum_{i=0}^{\lfloor \frac{4k+1}{4} \rfloor} 2^1.2^{4i}$$

$$= \sum_{i=0}^{\lfloor \frac{4k+1}{4} \rfloor} 2^{4i+1}$$

$$= 2^1 + 2^5 + ... + 2^{4.\lfloor \frac{4k+1}{4} \rfloor + 1}.$$

We want to prove that for $n = 4k + 5$, double edge-vertex domination number of 2-ary tree is

$$\gamma_{dev}(H^2_{4k+5}) = \sum_{i=0}^{\lfloor \frac{4k+5}{4} \rfloor} 2^{(4k+5)(mod 4)}.2^{4i}.$$

So,

$$\gamma_{dev}(H^2_{4k+5}) = \sum_{i=0}^{\lfloor \frac{4k+5}{4} \rfloor} 2^1.2^{4i}.$$

Since we add 4 levels to H^2_{4k+1}, the number of vertices of H^2_{4k+5} increases to 2^4 times of the vertices of the H^2_{4k+1} but we do operations in mod 4,

$$\gamma_{dev}(H^2_{4k+5}) \geq \gamma_{dev}(H^2_{4k+1}) + 2^{4\lfloor \frac{4k+1}{4} \rfloor + 1}.2^1$$
$$= 2^1 + 2^5 + ... + 2^{4\lfloor \frac{4k+1}{4} \rfloor + 1} + 2^{4\lfloor \frac{4k+1}{4} \rfloor + 1}.2^1$$
$$= 2^1 + 2^5 + ... + 2^{4\lfloor \frac{4k+1}{4} \rfloor + 1} + 2^{4\lfloor \frac{4k+1}{4} \rfloor + 1 + 1}$$
$$= 2^1 + 2^5 + ... + 2^{4\lfloor \frac{4k+1}{4} \rfloor + 1} + 2^{(4\lfloor \frac{4k+1}{4} \rfloor + \frac{4}{4}) + 1}$$
$$\geq 2^1 + 2^5 + ... + 2^{4\lfloor \frac{4k+1}{4} \rfloor + 1} + 2^{4\lfloor \frac{4k+1+4}{4} \rfloor + 1}$$
$$= 2^1 + 2^5 + ... + 2^{4\lfloor \frac{4k+1}{4} \rfloor + 1} + 2^{4\lfloor \frac{4k+5}{4} \rfloor + 1}$$
$$= 2^{(4k+5)(mod\ 4)}.2^0 + 2^{(4k+5)(mod\ 4)}.2^4 + ... + 2^{(4k+5)(mod\ 4)}.2^{4\lfloor \frac{4k+5}{4} \rfloor}$$
$$= \sum_{i=0}^{\lfloor \frac{4k+5}{4} \rfloor} 2^{(4k+5)(mod\ 4)}.2^{4i}.$$

- **Case 2:** For $n = 4k + 2$, $n\ mod\ 4 \equiv 2$, result is true for $n = 2$.

$$\gamma_{dev}(H^2_2) = \sum_{i=0}^{\lfloor \frac{2}{4} \rfloor} 2^{2(mod\ 4)}.2^{4i} = 2^2.2^0 = 2^2 = 4.$$

Our assumptation asserts that,

$$\gamma_{dev}(H^2_{4k+2}) = \sum_{i=0}^{\lfloor \frac{4k+2}{4} \rfloor} 2^{(4k+2)(mod\ 4)}.2^{4i}$$

$$= \sum_{i=0}^{\lfloor \frac{4k+2}{4} \rfloor} 2^2.2^{4i}$$

$$= \sum_{i=0}^{\lfloor \frac{4k+2}{4} \rfloor} 2^{4i+2}$$

$$= 2^2 + 2^6 + ... + 2^{4.\lfloor\frac{4k+2}{4}\rfloor+2}.$$

We want to prove that for $n = 4k+6$, double edge-vertex domination number of 2-ary tree is

$$\gamma_{dev}(H^2_{4k+6}) = \sum_{i=0}^{\lfloor\frac{4k+6}{4}\rfloor} 2^{(4k+6)(mod\ 4)}.2^{4i}.$$

So,

$$\gamma_{dev}(H^2_{4k+6}) = \sum_{i=0}^{\lfloor\frac{4k+6}{4}\rfloor} 2^2.2^{4i}.$$

Since we add 4 levels to H^2_{4k+2}, the number of vertices of H^2_{4k+6} increases to 2^4 times of the vertices of the H^2_{4k+2} but we do operations in mod 4,

$$\gamma_{dev}(H^2_{4k+6}) \geq \gamma_{dev}(H^2_{4k+2}) + 2^{4\lfloor\frac{4k+2}{4}\rfloor+2}.2^1$$

$$= 2^2 + 2^6 + ... + 2^{4\lfloor\frac{4k+2}{4}\rfloor+2} + 2^{4\lfloor\frac{4k+2}{4}\rfloor+2}.2^1$$

$$= 2^2 + 2^6 + ... + 2^{4\lfloor\frac{4k+2}{4}\rfloor+2} + 2^{4\lfloor\frac{4k+2}{4}\rfloor+2+1}$$

$$= 2^2 + 2^6 + ... + 2^{4\lfloor\frac{4k+2}{4}\rfloor+2} + 2^{4\lfloor\frac{4k+2}{4}\rfloor+1+2}$$

$$= 2^2 + 2^6 + ... + 2^{4\lfloor\frac{4k+2}{4}\rfloor+2} + 2^{(4\lfloor\frac{4k+2}{4}\rfloor+\frac{4}{4})+2}$$

$$\geq 2^2 + 2^6 + ... + 2^{4\lfloor\frac{4k+2}{4}\rfloor+2} + 2^{4\lfloor\frac{4k+2+4}{4}\rfloor+2}$$

$$= 2^2 + 2^6 + ... + 2^{4\lfloor\frac{4k+2}{4}\rfloor+2} + 2^{4\lfloor\frac{4k+6}{4}\rfloor+2}$$

$$= 2^{(4k+6)(mod\ 4)}.2^0 + 2^{(4k+6)(mod\ 4)}.2^4 + ... + 2^{(4k+6)(mod\ 4)}.2^{4\lfloor\frac{4k+6}{4}\rfloor}$$

$$= \sum_{i=0}^{\lfloor\frac{4k+6}{4}\rfloor} 2^{(4k+6)(mod\ 4)}.2^{4i}.$$

- **Case 3:** For $n = 4k+3$, $n\ mod\ 4 \equiv 3$ result is true for $n = 3$.

$$\gamma_{dev}(H^2_3) = \sum_{i=0}^{\lfloor\frac{3}{4}\rfloor} 2^{3(mod\ 4)}.2^{4i} = 2^3.2^0 = 2^3 = 8.$$

Our assumptation asserts that,

$$\gamma_{dev}(H^2_{4k+3}) = \sum_{i=0}^{\lfloor\frac{4k+3}{4}\rfloor} 2^{(4k+3)(mod4)}.2^{4i}$$

$$= \sum_{i=0}^{\lfloor\frac{4k+3}{4}\rfloor} 2^3.2^{4i}$$

$$= \sum_{i=0}^{\lfloor\frac{4k+3}{4}\rfloor} 2^{4i+3}$$

$$= 2^3 + 2^7 + ... + 2^{4.\lfloor\frac{4k+3}{4}\rfloor+3}.$$

We want to prove that for $n = 4k + 7$, double edge-vertex domination number of 2-ary tree is

$$\gamma_{dev}(H^2_{4k+7}) = \sum_{i=0}^{\lfloor \frac{4k+7}{4} \rfloor} 2^{(4k+7)(mod\ 4)}.2^{4i}.$$

So,

$$\gamma_{dev}(H^2_{4k+7}) = \sum_{i=0}^{\lfloor \frac{4k+7}{4} \rfloor} 2^3.2^{4i}.$$

Since we add 4 levels to $4k+3$, the number of vertices of $4k+7$ increases to 2^4 times of the vertices of the $4k+3$ but we do operations in mod 4,

$$\gamma_{dev}(H^2_{4k+7}) \geq \gamma_{dev}(H^2_{4k+3}) + 2^{4\lfloor \frac{4k+3}{4} \rfloor + 3}.2^1$$

$$= 2^3 + 2^7 + ... + 2^{4\lfloor \frac{4k+3}{4} \rfloor + 3} + 2^{4\lfloor \frac{4k+3}{4} \rfloor + 3}.2^1$$

$$= 2^3 + 2^7 + ... + 2^{4\lfloor \frac{4k+3}{4} \rfloor + 3} + 2^{4\lfloor \frac{4k+3}{4} \rfloor + 3 + 1}$$

$$= 2^3 + 2^7 + ... + 2^{4\lfloor \frac{4k+3}{4} \rfloor + 3} + 2^{4\lfloor \frac{4k+3}{4} \rfloor + 1 + 3}$$

$$= 2^3 + 2^7 + ... + 2^{4\lfloor \frac{4k+3}{4} \rfloor + 3} + 2^{(4\lfloor \frac{4k+3}{4} \rfloor + \frac{4}{4}) + 3}$$

$$\geq 2^3 + 2^7 + ... + 2^{4\lfloor \frac{4k+3}{4} \rfloor + 3} + 2^{4\lfloor \frac{4k+3+4}{4} \rfloor + 3}$$

$$= 2^3 + 2^7 + ... + 2^{4\lfloor \frac{4k+3}{4} \rfloor + 3} + 2^{4\lfloor \frac{4k+7}{4} \rfloor + 3}$$

$$= 2^{(4k+7)(mod\ 4)}.2^0 + 2^{(4k+7)(mod\ 4)}.2^4 + ... + 2^{(4k+7)(mod\ 4)}.2^{4\lfloor \frac{4k+6}{4} \rfloor}$$

$$= \sum_{i=0}^{\lfloor \frac{4k+7}{4} \rfloor} 2^{(4k+7)(mod\ 4)}.2^{4i}.$$

- **Case 4:** For $n = 4k$, $n\ mod\ 4 \equiv 0$, result is true for $n = 4$.

$$\gamma_{dev}(H^2_4) = 2 + \sum_{i=1}^{\frac{4}{4}} 2^{4i} = 2 + 2^4 = 18.$$

Our assumptation asserts that,

$$\gamma_{dev}(H^2_{4k}) = \sum_{i=1}^{\frac{4k}{4}} 2^{4i}$$

$$= \sum_{i=1}^{\frac{4k}{4}} 2^{4i}$$

$$= \sum_{i=1}^{\frac{4k}{4}} 2^{4i} = 2^1 + 2^4 + ... + 2^{4.\frac{4k}{4}}.$$

We want to prove that for $n = 4k + 4$, double edge-vertex domination number of 2-ary tree is

$$\gamma_{dev}(H^2_{4k+4}) = \sum_{i=1}^{\frac{4k+4}{4}} 2^{4i}.$$

Since we add 4 levels to H^2_{4k}, the number of vertices of H^2_{4k+4} increases to 2^4 times of the vertices of the H^2_{4k} but we are operating according to (mod 4),

$$\gamma_{dev}(H^2_{4k+4}) \geq \gamma_{dev}(H^2_{4k}) + 2^{4\frac{4k}{4}}.2^1$$

$$= 2^1 + 2^4 + ... + 2^{4\frac{4k}{4}} + 2^{4\frac{4k}{4}}.2^1$$

$$= 2^1 + 2^4 + ... + 2^{4\frac{4k}{4}} + 2^{4\frac{4k}{4}+1}$$

$$= 2^1 + 2^4 + ... + 2^{4\frac{4k}{4}} + 2^{(4\frac{4k}{4}+\frac{4}{4})} = 2^1 + 2^4 + ... + 2^{4\frac{4k}{4}} + 2^{4\frac{4k+4}{4}}$$

$$= \sum_{i=1}^{\frac{4k+4}{4}} 2^{4i}.$$

□

4 Conclusion

In graph theory, domination is one of those important concepts in the stability analysis of communication networks modelled by graphs. There are various types of domination depending on structure and properties of dominating sets. Domination has a wide range of applications. Both edge-vertex and vertex edge domination are concepts on which many researchers are working. Depending on the structure of network, sometimes any break down on links have more importance than center so we decided to work on edge sets. Then we define a new domination concept called as double edge-vertex domination. We obtain results on basic graph classes P_n, C_n,K_n,$K_{m,n}$ and on well know tree classses B_n, H^2_n and prove results. For future work we plan to generalize results on trees and find results under graph operations.

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